Realization of Topological Polynomials as Complex Polynomials

Adam Carty
The University of Alabama at Birmingham
acarty@uab.edu

In this talk, we explore a condition on the postcritical set of a branched covering map, f_t, of the Riemann sphere such that $f_t^{-1}(\infty) = \{\infty\}$, i.e. a *topological polynomial*, which guarantees the existence of a complex polynomial P such that f_t and P are topologically conjugate. This condition is that the postcritical set is such that for each critical point c, some iterate $f_t^n(c)$ $(n > 0)$ of c is a critical point. In this scenario, we show that the conditions of Thurston's Theorem are met. It follows that there exists a complex polynomial P which is *Thurston equivalent* to f_t. Thurston equivalence relates f_t and P via isotopic homeomorphisms. We use this relationship to show that f_t and P are topologically conjugate.

Every topological polynomial f_t has a combinatorial model described by a *lamination* in the unit disk. These models are used to study the dynamics of f_t, and vice versa, each lamination can be used to construct a topological polynomial on the Riemann sphere. Thus, we can study the properties of a given lamination (under appropriate conditions) and our main theorem gives rise to an actual complex polynomial with understood properties.