Bounded remainder sets for rotations on compact groups

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Bounded remainder sets for rotations on the s-torus, $\mathbb{T}^s = \mathbb{R}^s/\mathbb{Z}^s$, $s \geq 2$, have been widely studied since the 1920's and there now exists a complete classification of volumes of bounded remainder sets for toral rotations in any dimension. Subsequently, a classification of bounded remainder sets for rotations on an uncountable collection of connected, compact subgroups of the adelic torus $\mathbb{A}/\mathbb{Q}$ has been given.

In this joint work with Joanna Furno and Alan Haynes, we complete this direction of inquiry and give an explicit construction of polytopal bounded remainder sets of all possible volumes, for any ergodic rotation on the d-dimensional adelic torus $\mathbb{A}^d/\mathbb{Q}^d$, $d \geq 2$. We prove a necessary and sufficient condition for ergodicity of the rotation and show that ergodicity in this setting is equivalent to unique ergodicity. We also relate the existence of bounded remainder sets to the existence of dynamical coboundaries for the rotation map. Our construction involves ideas from dynamical systems and harmonic analysis on the adeles, as well as a geometric argument that reduces the existence argument to the case of an irrational rotation on the torus $\mathbb{R}^d/\mathbb{Z}^d$. We also verify that all allowable volumes are obtained by this construction."

(joint work with Joanna Furno, Alan Haynes)