Continuity of Følner averages I

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The notion of generic/mean points goes back to the seminal work of Krylov and Bogolyubov. The first to investigate the question of what happens when all points of a dynamical system are generic for some invariant measure seem to be Dowker and Lederer in 1964. As it turns out, combining this property with other topological regularity criteria yields measure-theoretic rigidity results of the dynamical system. For example, minimality of the system implies its unique ergodicity in this setting. Another natural topological criterion in place of minimality is to assume that the map, which assigns each point its invariant measure to which it is generic, is continuous. By several recent works by different authors, the following picture emerges for abelian group actions in this setting: each point is generic for some ergodic measure and even stronger, each orbit closure is uniquely ergodic. In my talk, I will show that this is no longer the case for general actions by topological amenable groups, providing concrete counter-examples involving the group of all orientation preserving homeomorphisms on the unit interval as well as the Lamplighter group.
(joint work with Gabriel Fuhrmann and Till Hauser)