Applying chain-level Poincaré duality to string topology

Kate Poirier
City University of New York - CityTech
`kpoirier@citytech.cuny.edu`

String topology studies algebraic structures that arise by intersecting loops, where a "loop" can mean something topological or algebraic. For example (on the topological side) on the homology of the free loop space of a closed, oriented manifold, there is a binary operation called the "loop product" and a unary operation called the "BV operator." These two operations together give the homology of the free loop space the structure of a "BV alge bra." Separately (on the algebraic side) in the presence of an algebraic version of Poincaré duality, there is product and BV operator on the Hochschild cohomology of this algebra. These operations give the Hochschild cohomology of the algebra the structure of a BV algebra as well. Further, when the algebra is the cochain algebra of a closed, oriented, simply connected manifold there is an isomorphism between its Hochschild cohomology and the homology of the free loop space of the manifold. While Cohen and Jones showed that this isomorphism respects the product structure, subsequent work of Menichi suggested that, in the case of the 2-sphere with mod 2 coefficients, it did not respect the BV operator. In this talk, we describe these operations and show that with an appropriate updated algebraic version of Poincaré duality for algebras—one involving higher homotopies—Hochschild cohomology can be given a BV operator that is, in fact, preserved by the isomorphism from the homol ogy of the free loop space of the 2-sphere with mod 2 coefficients. This is joint work with Thomas Tradler.
(joint work with Thomas Tradler)