The weak Ramsey property

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A celebrated result of Kechris, Pestov, and Todorcevic (2005) discovered a surprising correspondence between extreme amenability of the automorphism group of a highly symmetric (ultra-homogeneous) structure and a Ramsey-like property of the class of its finite substructures. A topological group is called *extremely amenable* if each of its continuous action on a compact Hausdorff space has a fixed point.

We extend the KPT correspondence to automorphism groups of so-called *generic objects*. These are mathematical structures represented as limits of the most complicated sequences of objects of a fixed base category. Here, a *sequence* is just a functor from the natural numbers. The automorphism group of a generic object admits a natural completely metrizable topology, therefore it is natural to look at its continuous actions. We show that this group is extremely amenable if and only if the base category has the weak Ramsey property, a far reaching generalization of the model-theoretic Ramsey property discovered by Kechris, Pestov, and Todorcevic.

Our setup is fairly general and can be applied in model theory, algebra, and topology. As an example, we shall discuss trees with strong embeddings, leading to generalized Ważewski dendrites.
(joint work with Adam Bartos, Tristan Bice, Keegan Dasilva Barbosa)