Universally Sacks-indestructible combinatorial families of reals

Lukas Schembecker
University of Vienna
lukas.schembecker@univie.ac.at

Given a combinatorial family $\mathcal{F}$ of reals, we say that a forcing $\mathbb{P}$ preserves $\mathbb{F}$ iff there are no intruders for $\mathcal{F}$ after forcing with $\mathbb{P}$. We also say that $\mathcal{F}$ is $\mathbb{P}$-indestructible. We prove that if a type of combinatorial family of reals is arithmetically definable, then every family of this type which is indestructible by the product of Sacks forcing $\mathbb{S}^{\aleph_0}$ is in fact universally Sacks-indestructible, i.e. it is indestructible by any countably supported iteration or product of Sacks-forcing of any length.

In order to obtain this theorem, we prove the following result: For every condition $p \in \mathbb{S}^{\aleph_0}$ in the product of Sacks-forcing the statement "p forces an arithmetical property of the generic sequence s_G" can be translated into an equivalent Π^1_3-statement.

We then verify that many different types of combinatorial families of reals considered in combinatorial set theory (e.g. mad families, maximal cofinitary groups, maximal independent families, ultrafilter bases, unbounded families, splitting families, ...) can be defined arithmetically, so that our theorem applies.

Finally, under CH we provide a unified construction of universally Sacks-indestructible witnesses for many different combinatorial families of reals. As a new result we obtain the existence of a universally Sacks-indestructible maximal cofinitary group.
(joint work with Vera Fischer)