Non-meager subsets of compact Hausdorff spaces

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Given a topological space X, how small can a non-meager subset of X be? For the real line, or (equivalently) any other Polish space without isolated points, the answer to this question is denoted $\mathrm{non}(\mathcal{M})$, one of the classic cardinal characteristics of the continuum. In this talk I will sketch a proof that for any completely metrizable space X with no isolated points, the smallest size of a non-meager subset of X, denoted $\mathrm{non}(\mathcal{M}_X)$, is exactly

$$\mathrm{non}(\mathcal{M}_X) = \mathrm{cf}[\kappa]^{\omega} \cdot \mathrm{non}(\mathcal{M}),$$

where κ is the minimum weight of a nonempty open subset of X. As a corollary, we also show that if X is a compact Hausdorff space X with π-weight κ, then $\mathrm{non}(\mathcal{M}_X) \leq \mathrm{cf}[\kappa]^{\omega} \cdot \mathrm{non}(\mathcal{M})$. In the compact Hausdorff case, however, this bound is consistently not sharp: it is possible that such a space has non-meager subsets of even smaller cardinality.